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STABILITY ANALYSIS OF COHESION PROPERTIES OF COOPERATIVE AGENTS WITH LIMITED SENSOR CAPABILITY

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Networked unmanned aerial vehicles (UAVs) are being developed for various applications. Multiple robots can be designed to coordinate to accomplish certain tasks. Their cooperative behaviors resemble, to a certain extent, those of bacteria, bees and birds that work together for food in the biological world. Suppose that we refer to all such groups of entities as "social foraging swarms." In order for such multi-agent systems to succeed it is often critical that they can both maintain cohesive behaviors and appropriate respond to environmental stimuli. In this paper we derive stability conditions under which social foraging swarms with limited sensing capability maintain cohesiveness when following certain resource profiles. The results are verified with simulations and challenge us to look for some connections between swarms with limited sensing capability, noisy measurements, and changes of communication topology.

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Stability Analysis of Cohesion Properties of Cooperative Agents with Limited Sensor Capability*

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Abstract

Networked unmanned aerial vehicles (UAVs) are being developed for various applications. Multiple robots can be designed to coordinate to accomplish certain tasks. Their cooperative behaviors resemble, to a certain extent, those of bacteria, bees and birds that work together for food in the biological world. Suppose that we refer to all such groups of entities as "social foraging swarms." In order for such multi-agent systems to succeed it is often critical that they can both maintain cohesive behaviors and appropriately respond to environmental stimuli. In this paper we derive stability conditions under which social foraging swarms with limited sensing capability maintain cohesiveness when following certain resource profiles. The results are verified with simulations and challenge us to look for some connections between swarms with limited sensing capability, noisy measurements, and changes of communication topology.

1 Introduction

Swarming has been studied extensively in biology [1, 2] and engineering applications including "intelligent vehicle highway systems," formation control for robots, aircraft, and cooperative control for uninhabited autonomous (air) vehicles, etc. [3, 4, 5, 6]. Early work on swarm stability is in [7, 8]. Some lately work includes [9, 10, 11], where the authors also consider asynchronous and time delays. Some previous work studies swarms that perform social foraging (i.e., follow certain resource profiles while achieving cohesiveness) [12, 13, 14, 15, 16]. Due to the limited sensing capability of the agents or broken communication network, changes on communication topology of the swarm agents may happen and affect the system stability. Some work on the topology changes in multi-agents includes [17, 18], where the authors study the convergence of the system based on graph theory.

In this paper, we continue some of our earlier work on studying stability properties of foraging swarms in [19, 20]. The main difference with our previous work is that here we consider the effect of the swarm agents with limited sensing capability. Thus, we are actually dealing with, to a certain extent, a dynamically changing network topology. Although it is not a fully switching network, the framework allows us to get some progress. Here, a fully switching network is approximated by adopting an appropriate sensor profile in our framework. We are able to obtain some local results which show an explicit relationship between the sensor profile and the initial condition of the system such that stable social foraging swarms may be achieved. In comparison, the authors in [18] deal with a fully switching network, but to obtain some stability results, they need to assume there exists an infinite sequence of contiguous, non-empty, bounded time intervals such that all the agents are linked together during each such interval. Such assumption may be difficult to verify, especially for a biological system.

The remainder of this paper is organized as follows: In Section 2 we introduce a basic model for agents, interactions, and the foraging environment. Then, a model for sensor profile, control, and error dynamics

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is introduced. Section 3 holds the main results on stability analysis of swarm cohesion. Section 4 holds the simulation results and some concluding remarks are provided in Section 5.

2 Models for the Cooperative Social Foraging Swarms

2.1 Basic Model for Agents, Interactions and Environment

Consider a swarm composed of an interconnection of N "agents," each of which has point mass dynamics given by

$$\begin{aligned} \dot{x}^i &= v^i \\ \dot{v}^i &= \frac{1}{M_i}u^i \end{aligned} \tag{1}$$

where $x^i \in \Re^n$ is the position, $v^i \in \Re^n$ is the velocity, M_i is the mass, and $u^i \in \Re^n$ is the (force) control input for the i^{th} agent.

Agent to agent interactions considered here are of the "attract-repel" type where each agent seeks to be in a position that is "comfortable" relative to its neighbors (and for us all other agents are its neighbors). Attraction indicates that each agent wants to be close to every other agent and it provides the mechanism for achieving grouping and cohesion of the group of agents. Repulsion provides the mechanism where each agent does not want to be too close to any other agent (e.g., for animals to avoid collisions and excessive competition for resources). Attraction here will be represented in u^i in a form like $-k\left(x^i - x^j\right)$ where $k > 0$ is a scalar that represents the strength of attraction. For repulsion, we adopt 2-norm and use a repulsion term in u^i of the form

$$k_r \exp\left(\frac{-\frac{1}{2}\|x^i - x^j\|^2}{r_s{}^2}\right)\left(x^i - x^j\right) \tag{2}$$

where $k_r > 0$ and $r_s > 0$. Other types of attraction and repulsion terms are also possible.

For the environment that the agents move in, we will simply consider the case where they move over a "resource profile" $J(x)$ where $x \in \Re^n$. Agents move in the direction of the negative gradient of $J(x)$ (i.e., in the direction of $-\nabla J(x) = -\frac{\partial J}{\partial x}$) in order to move away from "bad" areas and into "good" areas of the environment. We will assume that resource profile $J(x)$ is any profile that is continuous with finite slope at all points.

2.2 Model for Sensor Profile, Control and Error Dynamics

Let $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x^i$ and $\bar{v} = \frac{1}{N}\sum_{i=1}^{N} v^i$ be the centroid position and velocity of the swarm, respectively. To study the stability of swarm cohesion we study the dynamics of an error system with $e_p^i = x^i - \bar{x}$ and $e_v^i = v^i - \bar{v}$. Let $E^i = \left[{e_p^i}^\top, {e_v^i}^\top\right]^\top$ and $E = [{E^1}^\top, {E^2}^\top, \ldots, {E^N}^\top]^\top$. We have

$$\begin{aligned} x^i - x^j &= \left(x^i - \bar{x}\right) - \left(x^j - \bar{x}\right) = e_p^i - e_p^j \\ v^i - v^j &= \left(v^i - \bar{v}\right) - \left(v^j - \bar{v}\right) = e_v^i - e_v^j \end{aligned}$$

Define some short-hand notation as $x^{ij} = x^i - x^j$, $v^{ij} = v^i - v^j$, $e_p^{ij} = e_p^i - e_p^j$, $e_v^{ij} = e_v^i - e_v^j$. The error dynamics are given by

$$\begin{aligned} \dot{e}_p^i &= e_v^i \\ \dot{e}_v^i &= \frac{1}{M_i}u^i - \frac{1}{N}\sum_{j=1}^{N}\frac{1}{M_j}u^j \end{aligned} \tag{3}$$

Assume each agent has range-limited sensing capability. To represent this, define a monotonically decreasing function $f(x) : \Re^+ \rightarrow (0, 1]$ as the "sensor profile" such that $\lim_{x \to 0^+} f(x) = 1$ and $\lim_{x \to \infty} f(x) = 0$. A candidate function that satisfies these requirements is a Gaussian-shaped function centered at zero or

$$f(x) = -\frac{c_{f0}}{\pi}\arctan(c_{f1}(x - c_{f2})) + \frac{c_{f0}}{2} \tag{4}$$

where c_{f0} is a normalization factor and c_{f1} and c_{f2} may be adjusted to obtain different sensing range profiles. This function is shown in Figure 1 for one set of parameters.

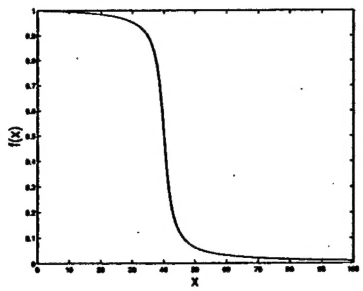

Figure 1: Sensor profile $f(x)$.

Let $N^i(t) = \sum_{j=1}^{N} f\left(\|x^{ij}\|\right)$ for all $i = 1, \ldots, N$. By definition $1 \leq N^i \leq N$ and N^i may or may not be an integer. Define for each agent i

$$\bar{x}^i = \frac{1}{N^i}\sum_{j=1}^{N} f\left(\|x^{ij}\|\right) x^j$$

$$\bar{v}^i = \frac{1}{N^i}\sum_{j=1}^{N} f\left(\|x^{ij}\|\right) v^j$$

which we view as swarm position and velocity centers from the perspective of agent i. From the equations above, the position and velocity of agent i relative to $\bar{x}^i$ and $\bar{v}^i$ are

$$\tilde{e}_p^i = x^i - \bar{x}^i = \frac{1}{N^i}\sum_{j=1}^{N} f\left(\|x^{ij}\|\right) x^i - \frac{1}{N^i}\sum_{j=1}^{N} f\left(\|x^{ij}\|\right) x^j = \frac{1}{N^i}\sum_{j=1}^{N} f\left(\|x^{ij}\|\right) x^{ij} \tag{5}$$

and similarly for $\tilde{e}_v^i$. The variable $\tilde{e}_p^i$ is the vector of displacements between the position of agent i and the center of all its neighbors where the notion of "neighbor" is specified via the sensor profile. We do *not* assume that x^{ij} (v^{ij}) and hence $\tilde{e}_p^i$ ($\tilde{e}_v^i$) can be sensed perfectly. In particular, let $d_p^{ij} \in \Re^n$ and $d_v^{ij} \in \Re^n$ be sensing errors. Let $\hat{x}^{ij} = x^{ij} - d_p^{ij}$ and $\hat{v}^{ij} = v^{ij} - d_v^{ij}$. Define $\hat{N}^i(t) = \sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right)$. We assume $d_p^{ii} \equiv d_v^{ii} \equiv 0$ for all i, so we have $1 \leq \hat{N}^i \leq N$. If in Equation (5) we replace x^{ij} (v^{ij}) with $\hat{x}^{ij}$ ($\hat{v}^{ij}$) we get

$$\hat{e}_p^i = \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right) \hat{x}^{ij} = \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right)\left(e_p^{ij} - d_p^{ij}\right) = e_p^i - \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right) e_p^j - \hat{d}_p^i \tag{6}$$

with $\hat{d}_p^i = \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right) d_p^{ij}$. Similarly we obtain $\hat{e}_v^i$, with $\hat{d}_v^i = \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\|\hat{x}^{ij}\|\right) d_v^{ij}$. Alternatively, we could also define $\hat{e}_p^i = \tilde{e}_p^i - d_p^i$ and get the same type of results as below. Here, we assume that we can measure $\hat{e}_p^i$ and $\hat{e}_v^i$.

We assume the resource profile is continuous with finite slope at all points, i.e., $\|\nabla J(x(t))\| \leq R$, where R is a known constant. Also assume the i^{th} agent senses $\nabla J\left(x^i(t)\right)$, the gradient of the profile at its position, but also with some error $d_f^i \in \Re^n$. That is, each agent i senses $\nabla J\left(x^i(t)\right) - d_f^i$. For simplicity, we will write $\nabla J\left(x^i(t)\right)$ as ∇J^i from now on.

For all the noise mentioned above, we assume they are sufficiently smooth and bounded by some constants. Specifically, we assume

$$\begin{aligned} \|d_p^{ij}\| &\leq D_p \\ \|d_v^{ij}\| &\leq D_v \\ \|d_f^i\| &\leq D_f \end{aligned} \tag{7}$$

where $D_p \geq 0$, $D_v \geq 0$ and $D_f \geq 0$ are known constants. Note that $\left\|\hat{d}_p^i\right\| \leq \frac{1}{\hat{N}^i}\sum_{j=1}^{N} f\left(\left\|\hat{x}^{ij}\right\|\right)\left\|d_p^{ij}\right\| = D_p$ and similarly, $\left\|\hat{d}_v^i\right\| \leq D_v$.

Now suppose the general form of the control input for each agent is

$$\begin{aligned} u^i &= -M_i k_p \hat{e}_p^i - M_i k_v \hat{e}_v^i - M_i k v^i - M_i k_f \nabla J^i \\ &+ M_i k_r \sum_{j=1, j\neq i}^{N} \exp\left(\frac{-\frac{1}{2}\left\|f\left(\left\|\hat{x}^{ij}\right\|\right)\hat{x}^{ij}\right\|^2}{{r_s}^2}\right)\left(f\left(\left\|\hat{x}^{ij}\right\|\right)\hat{x}^{ij}\right) \end{aligned} \tag{8}$$

where the scalars $k_p > 0$ and $k_v > 0$ indicate how aggressive each agent is in aggregating, $k > 0$ works as a "velocity damping gain," $k_r > 0$ sets how much that agent wants to be away from others, $r_s > 0$ represents its repulsion range, and $k_f > 0$ indicates that agent's desire to move along the negative gradient of the resource profile. To use Equation (8) as our control we assume that each agent knows its own velocity v^i, and $\hat{e}_p^i$ and $\hat{e}_v^i$ as discussed above. Also we assume that agent i can sense $f(\|\hat{x}^{ij}\|)\hat{x}^{ij}$ for all $j \neq i$, $j = 1, \ldots, N$. We think of $f(\|\hat{x}^{ij}\|)\hat{x}^{ij}$ as a noisy range-limited measurement of x^{ij}. In summary, in all cases each agent i only needs noisy range-limited sensing for its "decision making" via Equation (8).

Next, we derive the error system of which we will study stability properties. Let

$$\tau^i = \sum_{j=1, j\neq i}^{N} \exp\left(\frac{-\frac{1}{2}\left\|f\left(\left\|\hat{x}^{ij}\right\|\right)\hat{x}^{ij}\right\|^2}{{r_s}^2}\right)\left(f\left(\left\|\hat{x}^{ij}\right\|\right)\hat{x}^{ij}\right)$$

From Equations (6) and (8) we have

$$\begin{aligned} \dot{v}^i = \frac{1}{M_i}u^i &= -k_p\hat{e}_p^i - k_v\hat{e}_v^i - kv^i - k_f(\nabla J^i - d_f^i) + k_r\tau^i \\ &= -k_p e_p^i + \frac{k_p}{\hat{N}^i}\sum_{j=1}^{N} f\left(\left\|\hat{x}^{ij}\right\|\right)e_p^j - k_v e_v^i + \frac{k_v}{\hat{N}^i}\sum_{j=1}^{N} f\left(\left\|\hat{x}^{ij}\right\|\right)e_v^j - kv^i - k_f\nabla J^i \\ &\quad + k_r\tau^i + k_p\hat{d}_p^i + k_v\hat{d}_v^i + k_f d_f^i \\ &= -k_p e_p^i + \frac{k_p}{\hat{N}^i}\sum_{j=1}^{N}\left(1 - \Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\right)e_p^j - k_v e_v^i + \frac{k_v}{\hat{N}^i}\sum_{j=1}^{N}\left(1 - \Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\right)e_v^j - kv^i - k_f\nabla J^i \\ &\quad + k_r\tau^i + k_p\hat{d}_p^i + k_v\hat{d}_v^i + k_f d_f^i \\ &= -k_p e_p^i - \frac{k_p}{\hat{N}^i}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)e_p^j - k_v e_v^i - \frac{k_v}{\hat{N}^i}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)e_v^j - kv^i - k_f\nabla J^i \\ &\quad + k_r\tau^i + k_p\hat{d}_p^i + k_v\hat{d}_v^i + k_f d_f^i \\ &= -k_p e_p^i - k_v e_v^i - \frac{1}{\hat{N}^i}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\left([k_p,\ k_v]\,E^j\right) - kv^i - k_f\nabla J^i + k_r\tau^i + k_p\hat{d}_p^i + k_v\hat{d}_v^i + k_f d_f^i \end{aligned}$$

where $\Delta f\left(\left\|\hat{x}^{ij}\right\|\right) = 1 - f\left(\left\|\hat{x}^{ij}\right\|\right) \geq 0$ (recall $0 < f(x) \leq 1$). To obtain the above expression, we used the facts that $\sum_{j=1}^{N} e_p^j = 0$ and $\sum_{j=1}^{N} e_v^j = 0$. Then

$$\begin{aligned} \dot{\bar{v}} &= \frac{1}{N}\sum_{i=1}^{N}\dot{v}^i = \frac{1}{N}\sum_{i=1}^{N}\left[-k_p e_p^i - k_v e_v^i - \frac{1}{\hat{N}^i}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\left([k_p\ k_v]\,E^j\right) - kv^i - k_f\nabla J^i\right. \\ &\qquad \left. + k_r\tau^i + k_p\hat{d}_p^i + k_v\hat{d}_v^i + k_f d_f^i\right] \\ &= -\frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{\hat{N}^i}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\left([k_p\ k_v]\,E^j\right)\right] - k\bar{v} - k_f\bar{R} + k_r\bar{\tau} + k_p\bar{d}_p + k_v\bar{d}_v + k_f\bar{d}_f \end{aligned} \tag{9}$$

with $\bar{R}(t) = \frac{1}{N}\sum_{i=1}^{N} \nabla J^i(t)$, $\bar{\tau}(t) = \frac{1}{N}\sum_{i=1}^{N} \tau^i(t)$, $\bar{d}_p(t) = \frac{1}{N}\sum_{i=1}^{N} \hat{d}_p^i(t)$, $\bar{d}_v(t) = \frac{1}{N}\sum_{i=1}^{N} \hat{d}_v^i(t)$, and $\bar{d}_f(t) = \frac{1}{N}\sum_{i=1}^{N} d_f^i(t)$. So we have

$$\begin{aligned}
\dot{e}_v^i = \dot{v}^i - \dot{\bar{v}} &= -k_p e_p^i - (k_v + k)e_v^i - \frac{1}{\hat{N}^i}\sum_{j=1}^{N} \Delta f\left(\|\hat{x}^{ij}\|\right)\left([k_p\ k_v]\,E^j\right) \\
&\quad + \frac{1}{N}\sum_{l=1}^{N}\left[\frac{1}{\hat{N}^l}\sum_{j=1}^{N} \Delta f\left(\|\hat{x}^{lj}\|\right)\left([k_p\ k_v]\,E^j\right)\right] - k_f\left(\nabla J^i - \bar{R}\right) + k_r\left(\tau^i - \bar{\tau}\right) \\
&\quad + k_p\left(\hat{d}_p^i - \bar{d}_p\right) + k_v\left(\hat{d}_v^i - \bar{d}_v\right) + k_f\left(d_f^i - \bar{d}_f\right) \\
&= -k_p e_p^i - (k_v + k)e_v^i + \delta^i(E) + \phi^i
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
\delta^i(E) &= -\frac{1}{\hat{N}^i}\sum_{j=1}^{N} \Delta f\left(\|\hat{x}^{ij}\|\right)\left([k_p\ k_v]\,E^j\right) + \frac{1}{N}\sum_{l=1}^{N}\left[\frac{1}{\hat{N}^l}\sum_{j=1}^{N} \Delta f\left(\|\hat{x}^{lj}\|\right)\left([k_p\ k_v]\,E^j\right)\right] \\
\phi^i &= -k_f\left(\nabla J^i - \bar{R}\right) + k_r\left(\tau^i - \bar{\tau}\right) + k_p\left(\hat{d}_p^i - \bar{d}_p\right) + k_v\left(\hat{d}_v^i - \bar{d}_v\right) + k_f\left(d_f^i - \bar{d}_f\right)
\end{aligned}$$

With I an $n \times n$ identity matrix, the error dynamics of the i^{th} agent may be written as

$$\dot{E}^i = \overbrace{\begin{bmatrix} 0 & I \\ -k_p I & -(k_v + k)\,I \end{bmatrix}}^{A} E^i + \overbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}^{B} \left(\delta^i(E) + \phi^i\right) \tag{11}$$

Note that any matrix

$$\begin{bmatrix} 0 & I \\ -k_1 I & -k_2 I \end{bmatrix}$$

with $k_1 > 0$, $k_2 > 0$ has eigenvalues given by the roots of $\left(s^2 + k_2 s + k_1\right)^n$, which are in the strict left half plane. Thus, the matrix A above is Hurwitz with $k_p > 0$, $k_v > 0$ and $k > 0$.

3 Stability Analysis of Cohesive Social Foraging

To study the stability of the error dynamics, it is convenient to choose Lyapunov function for each agent

$$V_i\left(E^i\right) = {E^i}^\top P E^i \tag{12}$$

with $P = P^\top$ a $2n \times 2n$ matrix and $P > 0$ (a positive definite matrix). Then we have

$$\dot{V}_i = {E^i}^\top \underbrace{\left(PA + A^\top P\right)}_{-Q} E^i + 2{E^i}^\top PB\left(\delta^i(E) + \phi^i\right) \tag{13}$$

Note that when $Q = Q^\top$ and $Q > 0$, the unique solution P of $PA + A^\top P = -Q$ has $P = P^\top$ and $P > 0$ as needed.

Choose for the composite system

$$V(E) = \sum_{i=1}^{N} V_i(E^i)$$

where $V_i\left(E^i\right)$ is given in Equation (12). Then we have

$$\sum_{i=1}^{N}\left(\lambda_{min}(P)\left\|E^i\right\|^2\right) \le V(E) \le \sum_{i=1}^{N}\left(\lambda_{max}(P)\left\|E^i\right\|^2\right) \tag{14}$$

For now, assume $\frac{1}{\hat{N}^i}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\|[k_p\ k_v]\| \leq \tilde{a}$ for any i and j *at any* t, with $\tilde{a}$ some constant to be determined. Apparently, the smaller

$$\max_{i,j} \frac{1}{\hat{N}^i}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)$$

the smaller $\tilde{a}$ can be. In fact, $\tilde{a}$ reaches its minimum of 0 when $\Delta f\left(\left\|\hat{x}^{ij}\right\|\right) = 0$ for all i and j. Then we have

$$\left\|\delta^i(E)\right\| \leq \sum_{j=1}^{N} \frac{1}{\hat{N}^i}\Delta f\left(\left\|\hat{x}^{ij}\right\|\right)\left\|[k_p\ k_v]\,E^j\right\| + \frac{1}{N}\sum_{l=1}^{N}\left[\frac{1}{\hat{N}^l}\sum_{j=1}^{N}\Delta f\left(\left\|\hat{x}^{lj}\right\|\right)\left\|[k_p\ k_v]\,E^j\right\|\right] \leq \sum_{j=1}^{N} 2\tilde{a}\left\|E^j\right\| \quad (15)$$

It is seen that the function $F(\psi) = \exp\left(\frac{-\frac{1}{2}\|\psi\|^2}{r_s{}^2}\right)\|\psi\|$, with ψ any real vector, has a unique maximum value of $\exp(-\frac{1}{2})r_s$ which is achieved when $\|\psi\| = r_s$. Also $f(x) \leq 1$ for any $x \geq 0$. So $\|\tau^i\| \leq \exp(-\frac{1}{2})r_s$ and thus, $\|\tau^i - \bar{\tau}\| \leq 2\exp(-\frac{1}{2})r_s$. Note $\|\hat{d}^i_p - \bar{d}_p\| \leq 2D_p$. Similarly, $\|\hat{d}^i_v - \bar{d}_v\| \leq 2D_v$ and $\|\hat{d}^i_f - \bar{d}_f\| \leq 2D_f$. Also $\left\|\nabla J^i - \bar{R}\right\| \leq 2R$. Then for all i we have r_τ such that

$$\|\phi^i\| \leq r_\tau = 2k_f R + 2k_r \exp\left(-\frac{1}{2}\right) r_s + 2k_p D_p + 2k_v D_v + 2k_f D_f \quad (16)$$

Using the above equations and the fact that $\|B\| = 1$ we have

$$\begin{aligned}
\dot{V}(E) &= \sum_{i=1}^{N}\dot{V}_i(E^i) = \sum_{i=1}^{N}\left[-E^{i\top}QE^i + 2E^{i\top}PB\left(\delta^i(E) + \phi^i\right)\right] \\
&\leq \sum_{i=1}^{N}\left[-\lambda_{min}(Q)\left\|E^i\right\|^2 + 2\left\|E^i\right\|\lambda_{max}(P)\left(\left\|\delta^i(E)\right\| + \left\|\phi^i\right\|\right)\right] \\
&\leq \lambda_{min}(Q)\sum_{i=1}^{N}\left[-\left\|E^i\right\|^2 + \left\|E^i\right\|\sum_{j=1}^{N}4\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}\tilde{a}\left\|E^j\right\| + 2\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}r_\tau\left\|E^i\right\|\right] \quad (17)
\end{aligned}$$

The equation above indicates that smaller value of $\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}$ is desirable for achieving stability. Note that the fraction is minimized by choosing $Q = I$, thus, we replace it with $\beta_M = \frac{2\lambda_{max}(P|_{Q=I})}{\lambda_{min}Q|_{Q=I}} = \lambda_{max}(P|_{Q=I})$, where the explicit form of β_M may be obtained by solving the Lyapunov equation $A^\top P + PA = -I$ and we have

$$\beta_M = \frac{(k_p+1)^2 + (k_v+k)^2}{2k_p(k_v+k)} + \sqrt{\left(\frac{k_p^2 + (k_v+k)^2 - 1}{2k_p(k_v+k)}\right)^2 + \frac{1}{k_p^2}} \quad (18)$$

So Equation (17) now is written as

$$\begin{aligned}
\dot{V}(E) &\leq \sum_{i=1}^{N}\left[-\left\|E^i\right\|^2 + \left\|E^i\right\|\sum_{j=1}^{N}2\beta_M\tilde{a}\left\|E^j\right\| + \beta_M r_\tau\left\|E^i\right\|\right] \\
&= \sum_{i=1}^{N}\left[-\left\|E^i\right\|^2 + c\left\|E^i\right\| + \left\|E^i\right\|\sum_{j=1}^{N}\left(a\left\|E^j\right\|\right)\right] \quad (19)
\end{aligned}$$

with $c = \beta_M r_\tau$ and $a = 2\beta_M\tilde{a}$ are positive constants. Note that for any θ, $0 < \theta < 1$,

$$\begin{aligned}
-\left\|E^i\right\|^2 + c\left\|E^i\right\| &= -(1-\theta)\left\|E^i\right\|^2 - \theta\left\|E^i\right\|^2 + c\left\|E^i\right\| \\
&\leq -(1-\theta)\left\|E^i\right\|^2,\ \forall\ \left\|E^i\right\| \geq r \\
&= \sigma\left\|E^i\right\|^2 \quad (20)
\end{aligned}$$

where $r = \frac{c}{\theta}$ and $\sigma = -(1-\theta) < 0$. This implies that as long as $\|E^i\| \geq r$, the first two terms in (19) combined will give a negative contribution to $\dot{V}(E)$.

Next, we seek conditions under which $\dot{V}(E) < 0$. To do this, we consider the third term in (19) and combine it with the above results. Consider the general situation where some of the E^i are such that $\|E^i\| < r$ and others are not. Define sets

$$\Pi_O = \{i : \|E^i\| \geq r,\ i \in 1, \ldots, N\} = \left\{i_O^1,\ i_O^2, \ldots,\ i_O^{N_O}\right\}$$

and

$$\Pi_I = \{i : \|E^i\| < r,\ i \in 1, \ldots, N\} = \left\{i_I^1,\ i_I^2, \ldots,\ i_I^{N_I}\right\}$$

where N_O and N_I are the size of Π_O and Π_I, respectively. Also, $\Pi_O \bigcup \Pi_I = \{1, \ldots, N\}$ and $\Pi_O \bigcap \Pi_I = \phi$. Of course, we do not know the explicit sets Π_O and Π_I; all we know is that they exist. The explicit values in the sets clearly depend on time but we will allow that time to be arbitrary so the analysis below will be for all t. Obviously the system may switch back and forth between the cases of $N_O = 0$ and $N_O > 0$. But note that after the system has switched to $N_O = 0$ at certain moment, if it persists there, then the system is bounded, with $\|E^i\| \leq r$ for all i. If otherwise, then $N_O > 0$ will happen. So from now on, we assume $N_O > 0$ (but not necessarily persists there), that is, the set Π_O is non-empty. Then using analysis ideas from the theory of stability of interconnected systems [21] and using Equations (19) and (20), we have

$$\begin{aligned}
\dot{V}(E) \leq\ & \sum_{i\in\Pi_O} \sigma \|E^i\|^2 + \sum_{i\in\Pi_O} \left(\|E^i\| \sum_{j\in\Pi_O} a \|E^j\| \right) + \sum_{i\in\Pi_O} \left(\|E^i\| \sum_{j\in\Pi_I} a \|E^j\| \right) + \\
& \sum_{i\in\Pi_I} \left(-\|E^i\|^2 + c\|E^i\| \right) + \sum_{i\in\Pi_I} \left(a \|E^i\| \sum_{j\in\Pi_O} \|E^j\| \right) + \sum_{i\in\Pi_I} \left(\|E^i\| \sum_{j\in\Pi_I} a \|E^j\| \right)
\end{aligned}$$

Note for each fixed N_O, with the corresponding $N_I = N - N_O$ we can find constants $K_1(N_I)$, $K_2(N_I)$ and $K_3(N_I)$ such that

$$\begin{aligned}
K_1(N_I) &\geq \sum_{j\in\Pi_I} a \|E^j\| = \sum_{i\in\Pi_I} a \|E^i\| \\
K_2(N_I) &\geq \sum_{i\in\Pi_I} \left(-\|E^i\|^2 + c\|E^i\| \right) \\
K_3(N_I) &\geq \sum_{i\in\Pi_I} \left(\|E^i\| \sum_{j\in\Pi_I} a \|E^j\| \right)
\end{aligned} \tag{21}$$

In fact, the equations above are satisfied by choosing

$$\begin{aligned}
K_1(N_I) &= N_I a r \\
K_2(N_I) &= \frac{N_I}{4} \beta_M^2 r_\tau^2 \\
K_3(N_I) &= N_I^2 r^2 a
\end{aligned} \tag{22}$$

Then, we have

$$\begin{aligned}
\dot{V}(E) &\leq \sum_{i\in\Pi_O} \sigma \|E^i\|^2 + \sum_{i\in\Pi_O} \left(\|E^i\| \sum_{j\in\Pi_O} a \|E^j\| \right) + K_1 \sum_{i\in\Pi_O} \|E^i\| + K_2 + K_1 \sum_{j\in\Pi_O} \|E^j\| + K_3 \\
&= \sum_{i\in\Pi_O} \sigma \|E^i\|^2 + \sum_{i\in\Pi_O} \left(\|E^i\| \sum_{j\in\Pi_O} a \|E^j\| \right) + 2K_1 \sum_{i\in\Pi_O} \|E^i\| + K_2 + K_3
\end{aligned}$$

Let $w^\top = \left[\|E^{i_O^1}\|, \|E^{i_O^2}\|, \ldots, \|E^{i_O^{N_O}}\|\right]$ (the composition of this vector can be different at different times) and the $N_O \times N_O$ matrix $S = [s_{jn}]$ be specified by

$$s_{jn} = \begin{cases} -(\sigma + a), & j = n \\ -a, & j \neq n \end{cases} \tag{23}$$

so we have

$$\dot{V}(E) \leq -w^\top S w + 2K_1 \sum_{i \in \Pi_O} \|E^i\| + K_2 + K_3$$

It can be proven that the eigenvalues of matrix S include one at $-(\sigma + N_O a)$ and $N_O - 1$ repeated value of $-\sigma$. Since $a > 0$, we have $\lambda_{min}(S) = -(\sigma + N_O a)$. Assume $a < -\frac{\sigma}{N}$, then it is guaranteed that $a < -\frac{\sigma}{N_O}$ and thus, $\lambda_{min}(S) > 0$, so we have

$$\begin{aligned} \dot{V}(E) &\leq -\lambda_{min}(S) \sum_{i \in \Pi_O} \|E^i\|^2 + 2K_1 \sum_{i \in \Pi_O} \|E^i\| + K_2 + K_3 \\ &\leq (\sigma + N_O a) \sum_{i \in \Pi_O} \|E^i\|^2 + 2K_1 \sqrt{N_O \sum_{i \in \Pi_O} \|E^i\|^2} + K_2 + K_3 \\ &= (\sigma + N_O a) {E_O}^2 + 2K_1 \sqrt{N_O} E_O + K_2 + K_3 \end{aligned} \tag{24}$$

where $E_O = \sqrt{\sum_{i \in \Pi_O} \|E^i\|^2}$. Above we used the fact that given n real numbers, their mean value is smaller than or equal to their rms (root of mean square) value.

Now define function

$$F(\vartheta, N_I) = K_0(N_I)\vartheta^2 + 2K_1(N_I)\sqrt{N - N_I}\vartheta + K_2(N_I) + K_3(N_I) \tag{25}$$

with

$$K_0(N_I) = \sigma + (N - N_I)a \tag{26}$$

and $K_i(N_I)$ are defined in Equation (22) for $i = 1, 2, 3$. Assume ϑ is the maximum root of $F(\vartheta, N_I) = 0$ with N_I fixed. Then for any N_I, we have $F(\vartheta, N_I) < 0$ for $\vartheta > E_M$ with constant E_M defined as

$$E_M = \max_{0 \leq N_I < N} \vartheta$$

By comparing Equation (24) with (25), we can see that $\dot{V}(E) < 0$ whenever $E_O^2 = \sum_{i \in \Pi_O} \|E^i\|^2 > E_M^2$. From the definitions of Π_O, Π_I and r, we may further deduce that $\dot{V}(E) < 0$ whenever $\sum_{i=1}^N \|E^i\|^2 > E_M^2 + (N-1)r^2$. Let

$$E_\varepsilon = E_M^2 + (N-1)r^2 + \varepsilon \tag{27}$$

where ε is an arbitrarily small positive number. Also let $E_S = \sum_{i=1}^N \|E^i\|^2$ and define compact set $\Omega_B = \{E_S \mid E_S \leq E_\varepsilon\}$. Then $\dot{V}(E) < 0$ on $\partial\Omega_B$ (the boundary of Ω_B). Note that on $\partial\Omega_B$, from Equation (14) and (27) we have

$$\lambda_{min}(P) \sum_{i=1}^N \|E^i\|^2 \leq V(E) \leq \lambda_{max}(P) \sum_{i=1}^N \|E^i\|^2 = \beta_M E_\varepsilon$$

Since for any i it satisfies that $\|E^i\|^2 \leq \sum_{i=1}^N \|E^i\|^2$, we have $\lambda_{min}(P)\|E^i\|^2 \leq V(E) \leq \beta_M E_\varepsilon$ for any i, that is,

$$\max_i \|E^i\| \leq \sqrt{\frac{\beta_M}{\lambda_{min}(P)} E_\varepsilon} = \sqrt{\frac{\beta_M}{\beta_m} E_\varepsilon} \tag{28}$$

where $\beta_m = \lambda_{min}(P)$ and it can be solved that

$$\beta_m = \frac{(k_p+1)^2 + (k_v+k)^2}{2k_p(k_v+k)} - \sqrt{\left(\frac{k_p^2 + (k_v+k)^2 - 1}{2k_p(k_v+k)}\right)^2 + \frac{1}{k_p^2}}$$

Equation (28) gives an upper bound of $\|E^i\|$ for any i which no agent could go beyond if the system starts initially within the set Ω_B.

Finally, recall we assumed that $\frac{1}{N^i}\Delta f\left(\|\hat{x}^{ij}\|\right)\|[k_p\ k_v]\| \leq \tilde{a}$ for any i and j at any t. From Equation (28), this is justified if $E_S(0) \in \Omega_B$ and the sensor profile $f(x)$ is such that

$$f\left(2\sqrt{\frac{\beta_M}{\beta_m}E_\varepsilon} + \sqrt{D_p^2 + D_v^2}\right) \geq 1 - \frac{Na}{2\beta_M\sqrt{k_p^2 + k_v^2}} \tag{29}$$

since $\Delta f(x) = 1 - f(x)$, $a = 2\beta_M\tilde{a}$ and $\|\hat{x}^{ij}\| \leq \|E^i\| + \|E^j\| + \left\|\left[{d_p^{ij}}^\top, {d_v^{ij}}^\top\right]^\top\right\| < 2\max_i\|E^i\| + \sqrt{D_p^2 + D_v^2}$.

With all the deductions above, we collect all the conditions and state them in the following theorem.

Theorem 1. *Consider the N-agent error system described by the model in Equation (11). Assume the resource profile is continuous with finite slope at all points such that* $\|\nabla J(x(t))\| \leq R$. *Define* $r_\tau = 2k_f R + k_r\exp(-\frac{1}{2})r_s + 2k_pD_p + 2k_vD_v + 2k_fD_f$. *Let*

$$\beta_{M,m} = \frac{(k_p+1)^2 + (k_v+k)^2}{2k_p(k_v+k)} \pm \sqrt{\left(\frac{k_p^2 + (k_v+k)^2 - 1}{2k_p(k_v+k)}\right)^2 + \frac{1}{k_p^2}}$$

Also let θ *be some constant satisfies* $0 < \theta < 1$ *and* $r = \frac{1}{\theta}\beta_M r_\tau$. *Assume there exist positive constants* $a < \frac{1-\theta}{N}$ *and* $E_M = \max_{0\leq N_I < N}\vartheta$, *where* ϑ *is the maximum root of function*

$$F(\vartheta, N_I) = K_0(N_I)\vartheta^2 + 2K_1(N_I)\sqrt{N - N_I}\vartheta + K_2(N_I) + K_3(N_I)$$

with

$$\begin{aligned}
K_0(N_I) &= -(1-\theta) + (N - N_I)a \\
K_1(N_I) &= N_I a r \\
K_2(N_I) &= \frac{N_I}{4}\beta_M^2 r_\tau^2 \\
K_3(N_I) &= N_I^2 r^2 a
\end{aligned}$$

such that for the sensor profile we have

$$f\left(2\sqrt{\frac{\beta_M}{\beta_m}E_\varepsilon} + \sqrt{D_p^2 + D_v^2}\right) \geq 1 - \frac{Na}{2\beta_M\sqrt{k_p^2 + k_v^2}}$$

with $E_\varepsilon = E_M^2 + (N-1)r^2 + \varepsilon$ *and* ε *an arbitrarily small positive number. Define* $E_S(t) = \sum_{i=1}^N \left\|E^i(t)\right\|^2$ *and set* $\Omega_B = \{E_S \mid E_S \leq E_\varepsilon\}$. *If it satisfies* $E_S(0) \in \Omega_B$, *then the trajectories of the error system is uniformly bounded and* $\|E^i\| \leq \sqrt{\frac{\beta_M}{\beta_m}E_\varepsilon}$ *for all* i *and* t.

Remark. This theorem gives us a condition on the sensor profile and the initial condition of the system such that the system trajectories are bounded. If the agents start close enough to each other and the sensor profile is "flat" enough, then the swarm can stay cohesive while move along the resource profile. The results obtained here are quite conservative since we have to over bound many nonlinear terms in the deduction.

To reduce the demand to the "flat" range of the sensor, smaller E_ε is desirable. From Equation (27) and (29) we can see that smaller k_r and r_s are helpful since that means agents "push" each other less. Smaller k_f is also helpful since it means the agents may get distracted less by environment and thus, have better chance to stay cohesive. A "flat" resource profile, meaning smaller R, also helps the swarm stay cohesive.

Although at the beginning of the paper we assume all the agents will follow the same resource profile, in fact from the deduction of the theorem we can see that the result also holds for the case when the agents are following different profiles, so long as those profiles are continuous with finite slope at all points. For this case, the preservation of cohesiveness indicates that due to the desire to stay together, the agents each sacrifice following their own profile and compromise to follow certain "averaged" profile. This coincides the theoretical results we obtained in our previous works [19, 20] and is observed in the simulation results in the next section.

4 Simulation Results

In this section we show some simulation results. Unless otherwise stated, the parameters used are: $N = 10$, $k_p = 1$, $k_v = 1$, $k = 0.1$, $k_f = 0.1$, $k_r = 10$, and $r_s = 1$. The sensor profile we used is described in Equation (4) with $c_{f0} = 1.00015$, $c_{f1} = 0.5$, and $c_{f2} = 8252$. The noise bounds are $D_p = 5$, $D_v = 5$, and $D_f = 5$. For simplicity, instead of using the same (non-plane) profile, we assign different agents different plane resource profiles, with their gradients represented by randomly generated numbers. For the following simulation runs, the norm of the gradients are smaller than or equal to $R = 159$. We pick $\theta = 0.47$ and $a = 0.0472$. Then by solving Equation (25), we obtain $E_\varepsilon = 5.9 \times 10^6$ and $\sqrt{\frac{\beta_M}{\beta_m} E_\varepsilon} = 4116$, which specify the size of the set Ω_B and the upper bound of $\|E^i\|$ for all i, respectively, as stated in Theorem 1. The positions and velocity of the agents are initialized randomly. All simulations are run 20 seconds.

Figure 2 to 4 are for the case when the stability condition and initial condition specified by the theorem are satisfied. Figure 2 shows that the agents appear to move around erratically at the beginning, but soon they swarm together and move along the same direction, although they are assigned different plane resource profiles. Due to the effect of inter-agent repulsion, they do not shrink to one point but keep certain mutual spacing. From Figure 3 we can see that the agents have quite different velocities initially, but they gradually catch up with each other. Note this may not be the case if the profiles they are moving along are not plane profiles. That is, if we use some complicated profile (with "hills" and "valleys"), then their velocity may oscillate and will not always be the same. Figure 4 shows how the norms of the error of the agents changes as time goes by.

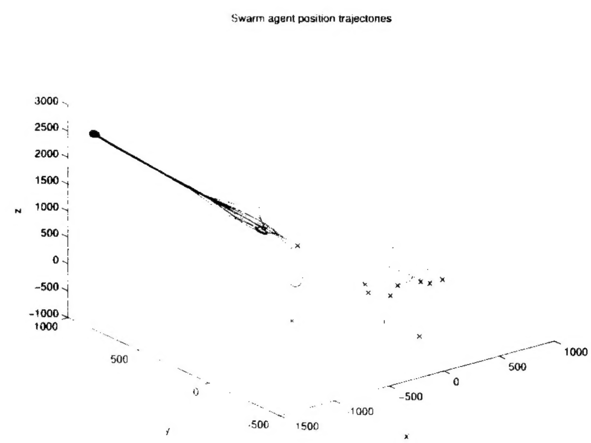

Figure 2: Position trajectories of the agents in 3d space (stable case).

If we keep all the parameters above the same but only decrease c_{f2}, then we reduce the capability of the sensor and unstable behaviors of the system are observed. Here we let $c_{f2} = 1500$ and Figure 5 shows that the swarm splits up and no more cohesiveness is achieved.

5 Concluding Remarks

In this paper we derive stability conditions under which social foraging swarms with limited sensing capability maintain cohesiveness when following certain resource profiles. It is interesting to note two points. Firstly, we can see some connection between the limited sensing capability and changes of communication topology. Specifically, if two agents are close to each other, then they can sense each other well and thus, are "connected." If they are far away, then each agent is located on a position that is at the "low end" of the sensor profile $f(x)$ of the other agent. Since we have $\lim_{x\to\infty} f(x) = 0$, then they are "disconnected." Apparently this change from being "connected" to being "disconnected" may be regarded as a change in the communication topology. Secondly, by comparing this paper with our previous work [19, 20], we can

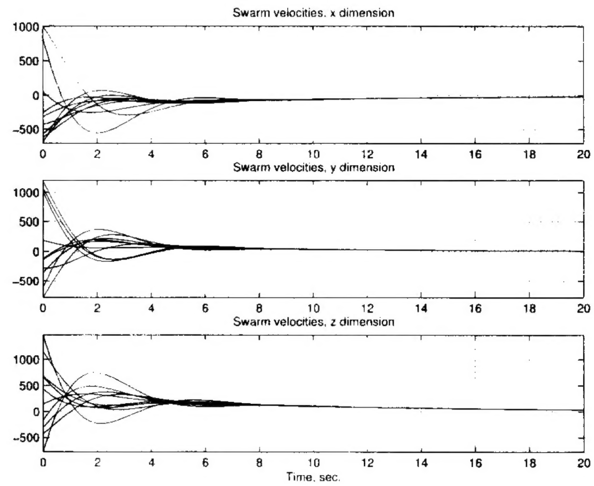

Figure 3: Velocity trajectories of the agents vs. Time (stable case).

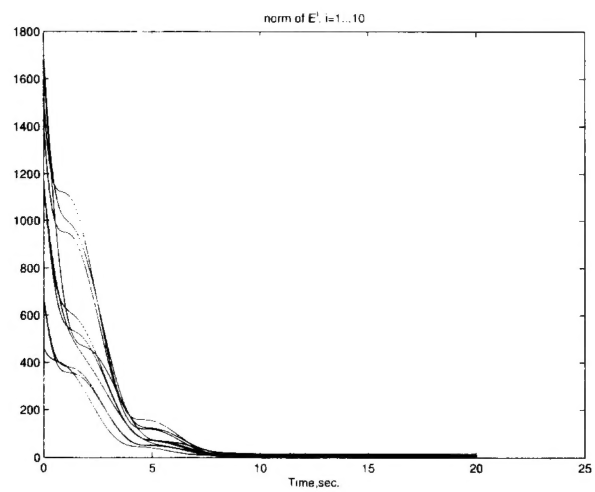

Figure 4: Norm of E^i of the agents vs. Time (stable case).

find some similarity in both the proof of the theory and the simulation results. This implies there may exist some connection between the limited sensing capability and a noisy environment/measurements. Now, if we combine the two points mentioned above, it would be interesting for us to think about the following question: Is it possible to draw some explicit connection between the two seemingly different topics of noisy environment/measurements and changes of communication topology? If we could do so, then it is possible to overcome some difficult problems on one topic by solving the "counterpart" (but possibly easier) problem in the other topic.

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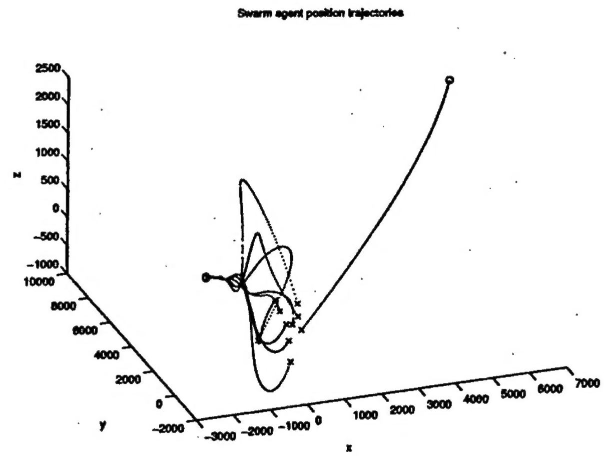

Figure 5: Position trajectories of the agents in 3d space (unstable case).

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